Indian J. Phys. **75A** (2), 175-180 (2001)

IJP A
an international journal

Non-singlet structure function at high x and neutrino scattering

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Received 14 March 2000, accepted 2 June 2000

Abstract We report an approximate solution of DGLAP equation approximated at high x for non-singlet structure function and comment on uniqueness problem. We apply the formalism to the CCFR neutrino structure function data of $xF_3(x, Q^2)$ and suggest a phenomenological form of it using Gross Llewellyn Smith Sum Rule Higher twist effects are also incorporated in the analysis

Keywords Structure function, high x, neutrino scattering

PACS Nos. 13 15 +g, 13 60.Hb, 12 38 Bx

1. Introduction

The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [1] are the basic tools to study the Q^2 evolution of the structure functions. In recent years, even at low x, these equations have been explored with impressive phenomenological success in the double asymptotic scaling limit [2]. Thus even though Balitsky-Fadin-Kuraev-Lipatov (BFKL) [3] or Gribov-Lipatov-Ryskin (GLR) [4] equations are theoretically more appealing at low x, DGLAP equations are simple perturbative tools which are relevant for the presently accessible $x - Q^2$ range of structure functions.

In recent years, approximate solutions of these equations have been pursued at low [5, 6] as well as at high [7, 9] x. In low x version of the formalism, Taylor series expansion was used to convert them into a set of partial differential equations in x (the Bjorken variable) and $t = \ln\left(\frac{Q^2}{\Lambda^2}\right)$.

One of the limitations of the approach is that the solutions reported are not unique [5, 6]. They are selected as the simplest ones with a single boundary condition–the non-perturbative x distribution at some initial point $t = t_0$. However, complete solution of DGLAP equations with two differential variables in general need two boundary conditions [10]. Besides the formalism was tested only in the basis of electron/muon-proton/deuteron data [11-15].

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Recently, the uniqueness problem for the non-singlet structure functions at low x has been reported [16]. The aim of the present paper is to make a similar analysis in the high x domain and apply it in neutrino structure function $xF_3(x, Q^2)$ [17-20].

The approximate solution of DGLAP equations valid at high x ($x \geq 0.1$) had been formulated [7]. Later on, alternative approximate solutions were also reported [9]. While the former solution was obtained from plausible form of trial solutions and then shown to have validity for $x \geq 0.1$ at the SLAC-MIT range [11] phenomenologically, the later one was by construction valid only for $x \geq .47$. It will be worthwhile to study the relative merits of these two in the analysis of SF_3. This is also an aim of the present paper.

The Gross-Llewellyn Smith (GLS) Sum Rule provides one of the important constraints on the structure function $xF_3(x, Q^2)$. In recent years, there have been intense experimental [17-20] and theoretical works [22-24] on the GLS sum rule. In this paper, we will use the sum rule to suggest the Q^2-evolution of $xF_3(x, Q^2)$ in the entire x-range.

A number of recent papers have discussed power corrections to the non-singlet structure functions and sum rules using the language of infrared renormalons [25-27]. Phenomenologically, it appears that the corrections computed in this way provide a good guide to the form of the higher twist (HT) contributions observed experimentally. Higher twist contributions to xF_3 have also been estimated phenomenologically [28] which we will incorporate in our result.

In Section 2 we develop the formalism while Section 3 contains the results. Section 4 is devoted to the conclusion and comments.

2. Formalism

2.1 Non-singlet structure function at high x :

The DGLAP equation for non-singlet structure function which evolve independent of singlet and gluon distributions can be written with the explicit form of splitting kernel in the first order of α, as

$$\frac{\partial F^{NS}}{\partial t}=\frac{A_f}{t}\Big[(3+4\log(1-x))F^{NS}(x,t)$$
$$+2\int_x^1\frac{dz}{1-z}\left((1+z^2)\,F^{NS}\left(\frac{x}{z},t\right)-2F^{NS}(x,t)\right)\Big], \quad (1)$$

where $t=\log\left(\frac{Q^2}{\Lambda^2}\right)$ and $A_f=\frac{4}{33-2N_f}$, N_f being the number of quark flavours.

Let us introduce the variable $u=1-z$ and note that

$$\frac{x}{1-u}=x\sum_{k=0}^{\infty}u^k. \quad (2)$$

The above series is convergent for $|u|<1$. Since $x<z<1$, so $0<u<1-x$ and hence the convergence condition is satisfied. Using (2), we write in (1)

$$F^{NS}\left(\frac{x}{z},t\right)=F^{NS}(x,t)+\sum_{l=1}^{\infty}\frac{x^l}{l!}\left(\sum_{k=1}^{\infty}u^k\right)^l\frac{\partial^l F^{NS}(x,t)}{\partial x^l}, \quad (3)$$

which covers the whole range of u, $0<u<1-x$.

Eq. (3) contains two infinite series – one in x and the other in u. As $x\to 1$, $u\to 0$ so that it is reasonable to write

$$F^{NS}\left(\frac{x}{z},t\right)=F^{NS}(x,t)+\sum_{l=1}^{\infty}\frac{x^l}{l!}u^l\frac{\partial^l F^{NS}(x,t)}{\partial x^l}. \quad (4)$$

In (4), the small expansion parameter is xu rather than x. Assuming that the higher order derivatives of non-singlet structure functions are non-singular as $x\to 1$, (4) can further be reduced to

$$F^{NS}\left(\frac{x}{z},t\right)=F^{NS}(x,t)+xu\frac{\partial F^{NS}(x,t)}{\partial x}. \quad (5)$$

Putting (5) in (1) and performing the integration, one has

$$Q(t)\frac{\partial F^{NS}(x,t)}{\partial t}+P(x)\frac{\partial F^{NS}(x,t)}{\partial x}=R(x,t,F^{NS}), \quad (6)$$

where $Q(t)$, $R(x,t,F^{NS})$ and $P(x)$ are defined as

$$Q(x,t)=t, \quad (7)$$

$$R(x,t,F^{NS})=R'(x)\,F^{NS}(x,t), \quad (8)$$

with $R'(x)=A_f[3+4\log(1-x)+(x-1)(x+3)]$ (9)

and $P(x)=\frac{2}{3}A_f x(x-1)(x^2+x+4)$. (10)

The general solution of (6) is obtained by solving the following auxiliary systems of ordinary differential equations

$$\frac{dx}{P(x)}=\frac{dt}{Q(t)}=\frac{dF^{NS}(x,t)}{R(x,t,F^{NS}(x,t))}. \quad (11)$$

If $u(x,t,F^{NS})=C_1$ (12)

and $v(x,t,F^{NS})=C_2$ (13)

are two independent solutions (C_1, C_2 being arbitrary constants) of (11), then general solution of (6) is

$$f(u,v)=0, \quad (14)$$

where f is an arbitrary function of u and v.

The auxiliary system (11) has three equations :

$$\frac{dx}{P(x)}=\frac{dt}{Q(t)}, \quad (15)$$

$$\frac{dx}{P(x)}=\frac{dF^{NS}}{R(x,t,F^{NS})} \quad (16)$$

and $\frac{dt}{Q(t)}=\frac{dF^{NS}}{R(x,t,F^{NS})}$. (17)

Solutions of (15) and (16) are

$$u(x,t,F^{NS})=t\,X^{NS}(x), \quad (18)$$

$$v(x,t,F^{NS})=F^{NS}(x,t)\,Y^{NS}(x), \quad (19)$$

with $X^{NS}=\exp\left[-\int\frac{dx}{P(x)}\right]$ (20)

and $Y^{NS}(x)=\exp\left[\int\frac{dx\,R'(x)}{P(x)}\right]$. (21)

Solution of (17) is in general not possible since it requires the additional information of explicit x and t dependence of $F^{NS}(x,t)$.

The general solution of (6) linear in $F^{NS}(x,t)$ is

$$u(x,t) + \alpha\, v(x,t,F^{NS}) = \beta, \tag{22}$$

α, β being arbitrary constants.

The physically plausible boundary conditions for non-singlet structure functions are

$$F^{NS}(x,t) = F^{NS}(x,t_0) \tag{23}$$

for some low $t = t_0$ and

$$F^{NS}(1,t) = 0 \tag{24}$$

for any t. While the first one corresponds to a non-perturbative input at some low momentum transfer, the second one corresponds to the expected large x ($x \to 1$) behaviour of any structure function (singlet as well as non-singlet) at any scale of momentum transfer consistent within the constituent counting rules [29, 30].

Using the boundary condition of (23) and (24) in (22), we have

$$t_0 X^{NS}(x) + \alpha F^{NS}(x,t_0) Y^{NS}(x) = \beta \tag{25}$$

and $\quad t X^{NS}(1) = \beta, \qquad (26)$

which leads to

$$F^{NS}(x,t) = \left(\frac{t}{t_0}\right) F^{NS}(x,t_0) \frac{X^{NS}(1) - X^{NS}(x)}{\frac{t}{t_0} X^{NS}(1) - X^{NS}(x)} \tag{27}$$

Explicit form of $X(x)$ is

$$X(x) = \exp\left[\left(\sqrt{(3/5)}\, Arctan \frac{1+2x}{\sqrt{15}} - \log(1-x) - \frac{3\log(x)}{4} - \frac{\log(x^2+x+4)}{16} \right) \right] \tag{28}$$

The auxiliary eq. (14) can also have a solution $\sim u^{-1}(x,t,F^{NS})$ instead of $u(x,t)$ defined in (17). In that case, using the same procedure as earlier, we will have

$$F^{NS}(x,t) = \left(\frac{t_0}{t}\right) F^{NS}(x,t_0) \frac{X(1)^{-1} - X(x)}{\frac{t_0}{t} X(1)^{-1} - X(x)^{-1}} \tag{29}$$

however $X(x)$ is singular at $x = 1$ due to (28). As a result, the solution (27) is physically unacceptable in $x \to 1$ limit.

On the other hand, (29) yields

$$F^{NS}(x,t) = \left(\frac{t_0}{t}\right) F^{NS}(x,t_0), \tag{30}$$

which is the unique prediction of t-evolution at high x within the present formalism.

2.2 Previous models :

In this section, we summarise the approximate form of non-singlet structure functions of Refs. [7] and [8].

Assuming the factorizability of structure function $F^{NS}(x, t)$ in x and t, solution of eq. (1) has the structure [7]

$$F^{NS}(x,t) = F^{NS}(x,t_0)(t/t_0)^{A_f[3+4\ln(1-x)+2I(x)]}. \tag{31}$$

where

$$I(x) = \int \frac{dz}{1-z} \left[(1+z^2) \frac{F^{NS}(x/z)}{F^{NS}(x)} - 2 \right] \tag{32}$$

Later [8], it was shown that if the trial solution is assumed to have the form

$$F^{NS}(x,t) = F^{NS}(x,t_0)\left(\frac{t}{t_0}\right)^{\beta(x)} \tag{33}$$

the exponent of (t/t_0) occurred in (31) automatically follows without the necessity of factorization hypothesis.

More recently [9], in high x approximation ($x \to 1$), the contribution from the integral $I(x)$ defined in (32) and occurred in (31) is neglected leading to the solution

$$F^{NS}(x,t) = F^{NS}(x,t_0)\left(\frac{t}{t_0}\right)^{A_f[3+4\ln(1-x)]} \tag{34}$$

Thus while (29) predicts fall of non-singlet structure function with increasing t, independent of specific x, eq. (34) on the other hand, predicts such a fall with t only for $x \leq 0.47$. We will make phenomenological comparison of all these findings.

2.3 Application to neutrino scattering :

In terms of constituent quarks [29], the non-singlet structure function xF_3 occurred in neutrino scattering is

$$xF_3(x,Q^2) = x\left[u_v(x,Q^2) + d_v(x,Q^2)\right], \tag{35}$$

where

$$xF_3(x,Q^2) = xF_3^{\nu P}(x,Q^2) + xF_3^{\bar{\nu} P}(x,Q^2), \tag{36}$$

u_v and d_v being the valence quark distributions inside the proton. xF_3 satisfies the evolution equation (1) and hence its approximate solutions (29), (31), (33) and (34). We will study these solutions in context to neutrino data [17-20].

The Gross-Llewellyn-Smith (GLS) Sum Rule [21] predicts the integral of xF_3 weighted by $1/x$, equals to the number of valence quarks inside the nucleon – three in naive quark model. With leading order QCD correction, the GLS Sum Rule reads [6]

$$S_{GLS} = \int_0^1 \frac{dx}{x} xF_3(x, Q^2) = 3\left[1 - \frac{3A_f}{t}\right], \tag{37}$$

which can be used to extrapolate our results beyond its large and small x regimes.

3. Results

The x-range explored in Columtia-Chicago-Fermi Lab-Rochester (CCFR) experiments [17-20] is outside the small x regime where the non-singlet structure function shows logarithmic Q^2 dependence ($F^{NS} \sim (t/t_0)$) [16]. We therefore, confine ourselves to the analysis of the CCFR data with the formalism developed in Section 2 for high x.

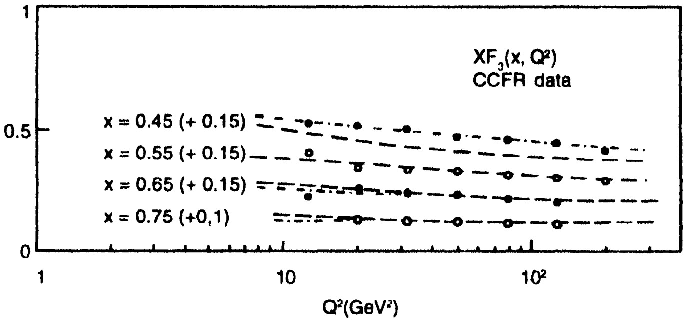

Figure 1. $xF_3^{\nu N}$ vs Q^2 for $x \geq 0.45$ using (30) of the text (dashed curves) and the exact result (dot-dashed curves) with MRST input (Ref. [31])

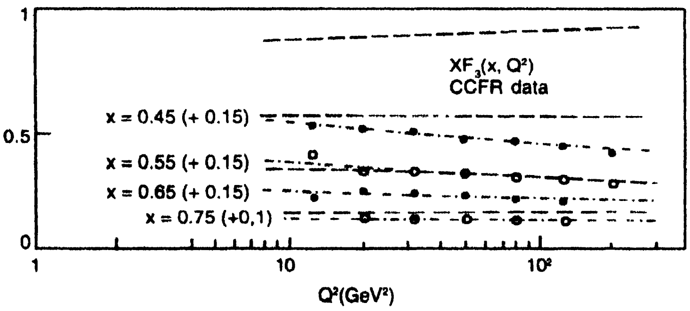

Figure 2. $xF_3^{\nu N}$ vs Q^2 for $x \geq 0.45$ using (31) of the text (dashed curves) Dot-dashed curves represent exact result. Inputs are similar to Figure 1.

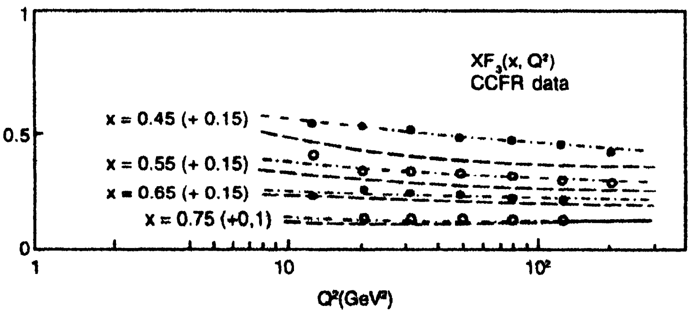

Figure 3. $xF_3^{\nu N}$ vs Q^2 for $x \geq 0.45$ using (34) of the text (dashed curves). Dot-dashed curves has meaning similar to Figure 1. Input and data are as in Figure 1.

In Figure 1, 2 and 3, we plot xF_3 vs Q^2 for $x \geq 0.45$ using (30), (31) and (34) of the text respectively (dashed curves) and compare with the exact results (dot-dashed curves) with MRST input [31]. A comparison of these figures show that the prediction of (34) [Figure 2] invariably overshoots the data while that of (31) [Figure 3] give reasonable agreement for higher x ($x \geq 0.75$). For (30) [Figure 1] agreement is better for $x \geq 0.55$. It suggests that eq. (30) is phenomenologically more viable compared with the other two forms (31) and (34). Invariably, it is also nearer to the exact results, not (31).

Let us now study the effects of higher twist terms using the analysis of Ref. [28]. To that end, we define

$$R_{HT} = \frac{xF_3^{HT}}{xF_3^{LT}} \cdot 100, \tag{38}$$

and plot it in Figures 4(a, b), which show that percentage of higher twist effects [28] of the CCFR data using eq. (30) for the leading twist term. It shows that for $Q^2 \sim 10$ GeV^2, $x = 0.65$ [Figures 4a]–HT can be as large as 46% while for $Q^2 \sim 100$ GeV and for the same value of x [Figure 4b] it becomes as low as 6%. On the other hand, for lower x, $x = 0.45$, HT effect in the same Q range varies only between 4%–5%.

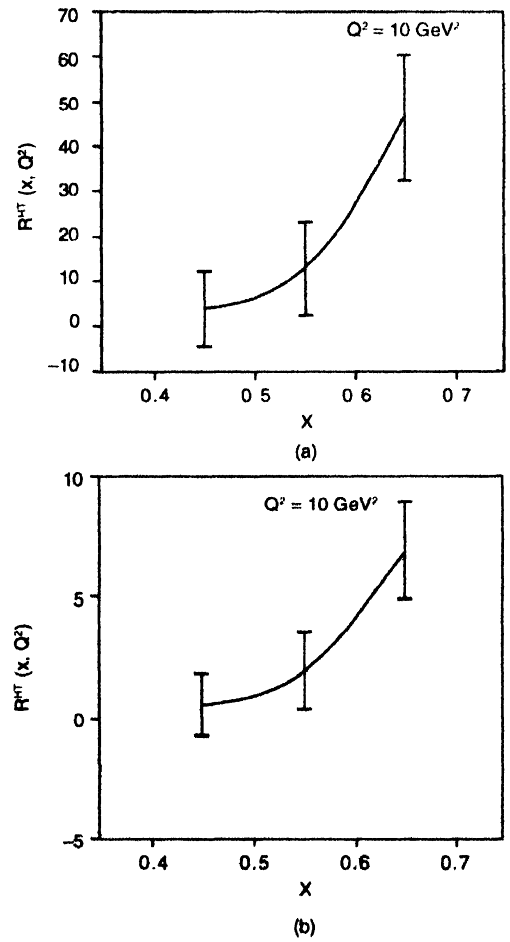

Figure 4. $R(x, Q^2)$ vs x as defined in (38) of the text for Q^2 = (a) 10 GeV^2 and (b) 100 GeV^2 respectively.

We now comment on the application of the GLS sum rule in the present analysis. In order to get information about the structure function xF_3 over the entire x-range from the GLS sum rule (37), let us divide the x-range into three regimes-low $(0 < x < a)$, intermediate $(a < x < b)$ and high $(b < x < 1)$, so that (37) yield

$$\int_0^a \frac{xF_3(x,t)}{x} + \int_a^b \frac{xF_3(x,t)}{x} + \int_b^1 \frac{xF_3(x,t)}{x} = 3\left[1 - \frac{3A_f}{t}\right] \tag{39}$$

Assuming the validity of $F^{NS} \sim (t/t_0)$ [16] and $F^{NS} \sim (t_0/t)$ is low x and high x regimes respectively, the following form of the structure function appears reasonable

$$F_3(x,t) = A(x) + B(x)\left(\frac{t}{t_0}\right) + C(x,t)\left(\frac{t_0}{t}\right) \tag{40}$$

with

$$\int_a^b A(x)\,dx = 3\,, \tag{41}$$

$$\int_a^b B(x)\,dx = -\int_0^a F_3(x,t_0)\,dx, \tag{42}$$

and $$\int_a^b C(x)\,dx = -\left[\int_b^1 F_3(x,t_0)\,dx + \frac{9A_f}{t}\right]. \tag{43}$$

Besides these, $A(x)$, $B(x)$ and $C(x)$ should also satisfy the following conditions

$$A(x) \to 0 \text{ for } b < x < a\,, \tag{44}$$

$$B(x) \to 0 \text{ for } x > b\,, \tag{45}$$

$$C(x,t) \to 0 \text{ for } x < a\,. \tag{46}$$

To find the exact forms of $A(x)$, $B(x)$ and $C(x,t)$ defined in (40) with conditions (44)-(46) are beyond the scope of present perturbative analysis. Moreover to do this, one also needs the exact numerical values of a and b as occurred in (39). From CCFR data we can only roughly infer that $a < 0.0075$ while $b \simeq 0.55$.

4. Conclusion and comments

In this paper, we have proposed the approximate form of solution of DGLAP equations at high x and applied the formalism to an analysis of the neutrino structure function $xF_3(x, Q^2)$ at high x. We have commented on the uniqueness of the solutions. Using Gross Llewellyn Smith sum rule, we have also suggested a phenomenological form valid in the entire x range. Higher twist effects have also been analysed in the present work. From phenomenological point of view, the suggested form (30) seems to fair better than (31) and (34) suggested earlier.

It is however, interesting to note that the known asymptotics of DGLAP equations (34) conform to the present one (30) at

$$A_f\,[3 + 4\ln(1-x)] = -1. \tag{47}$$

It corresponds to $x = 0.90$ with $N_f = 4$. Both the predictions differ within 10% in the range $x \sim 0.88 - 0.92$.

Let us now make a few comments. In recent years, there are analysis of xF_3 at the next to leading order (NLO) as well as next to next to leading order (NNLO) [32-34]. In the light of such a progress, it is meaningful to conclude with the physics issue, clearly brought out in the present work. We have shown that the asymptotics of DGLAP equation depends crucially on the boundary conditions, so much so that alternative asymptotics other than the traditionally known one (34) can be obtained, which conform to data as well as to exact results in a reasonable way.

Acknowledgments

One of us (DKC) is grateful to Prof A. Sidorov of JINR, Dubna, Russia for suggesting the analysis of neutrino scattering within the present scheme while the other (AD) acknowledges helpful discussions with Prof K. D. Krori. Financial support from the Department of Science & Technology, New Delhi is gratefully acknowledged.

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